MECHANICS

[Numbers 307 and 308 were omitted in the May issue.]

307. Proposed by LAENAS G. WELD, Pullman, Illinois.

Four forces W, X, Y, and Z, concurrent in O, are in equilibrium. Prove that

$$W : X : Y : Z :: \Delta_1 : \Delta_2 : \Delta_3 : \Delta_4,$$

where

$$\Delta_1 = \begin{vmatrix} 1 & \cos XOY & \cos XOZ \\ \cos XOY & 1 & \cos YOZ \\ \cos XOZ & \cos YOZ & 1 \end{vmatrix}^{\frac{1}{2}}; \quad \Delta_2 = \begin{vmatrix} 1 & \cos WOY & \cos WOZ \\ \cos WOY & 1 & \cos YOZ \\ \cos WOZ & \cos YOZ & 1 \end{vmatrix}^{\frac{1}{2}};$$

$$\Delta_3 = \begin{vmatrix} 1 & \cos WOX & \cos WOZ \\ \cos WOX & 1 & \cos XOZ \\ \cos WOZ & \cos XOZ & 1 \end{vmatrix}^{\frac{1}{2}}; \quad \Delta_4 = \begin{vmatrix} 1 & \cos WOX & \cos WOY \\ \cos WOX & 1 & \cos XOY \\ \cos WOY & \cos XOY & 1 \end{vmatrix}^{\frac{1}{2}}.$$

308. Proposed by H. S. UHLER, Yale University.

Prove that when a ray of light passes obliquely through a prism in such a manner as to maintain a constant value for the total deviation of the projection of the ray on a principal section, the ray inside the prism generates a cone of elliptical right section. It is assumed that the prism is surrounded by a medium having a smaller index of refraction than the index of the material of the prism.

NUMBER THEORY.

234. Proposed by FRANK IRWIN, University of California.

Start with any number, for instance 89, and add to it the number obtained by reversing the order of its digits: $89 + 98 = 187$. Now perform the same operation on the result: $187 + 781 = 968$. If we continue in this way we arrive, after a certain number of operations, at a number which reads the same forwards and backwards (24 operations bring us to 8813200023188). Will this be the case no matter with what number we start?

Note.—I am told that this is an old problem, but do not know whether it has ever been solved. (No other number under 100, except, of course, 98, requires so many operations to lead to the desired result, as 89 does.)

SOLUTIONS OF PROBLEMS.

ALGEBRA.

409. Proposed by C. E. GITHINS, Wheeling, W. Va.

Find integral values for the edges of a rectangular parallelepiped so that its diagonal shall be rational.

(*A*) REMARKS BY ARTEMAS MARTIN, Washington, D. C.

I. On page 162 of the May MONTHLY, Mr. Eells says: "I fail to see, however, how I solved a *different* problem from the one porposed."

The problem proposed is stated above; the problem solved by Mr. Eells follows:

Find integral values for the edges of a rectangular parallelepiped so that its diagonal, and the diagonal of one of its faces, shall be rational.

The problem proposed does not require the diagonal of one of the faces of the solid to be rational, and the added condition restricts its generality and limits the number of possible solutions.

II. Mr. Eells says on page 269 of the October issue, without any qualifying condition: "This gives the smallest rational parallelepiped," edges 3, 4, 12 and diagonal 13.